Evaluation of Hierarchical Homogenization on Gassmann Equation and Analysis of Generated Superresolution Digital Rocks

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Outline

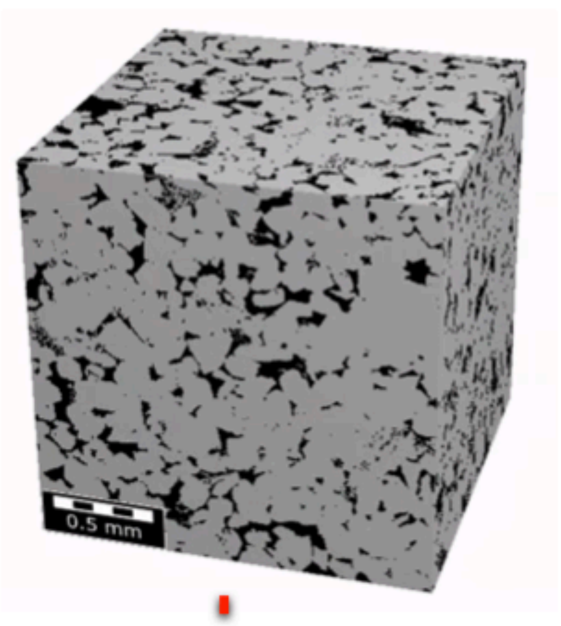
- General method of HHM and workflow
- Benchmarking HHM with Gassmann Equation
- Analysis of sub-cube mechanics
- Statistical analysis of GAN generated rocks

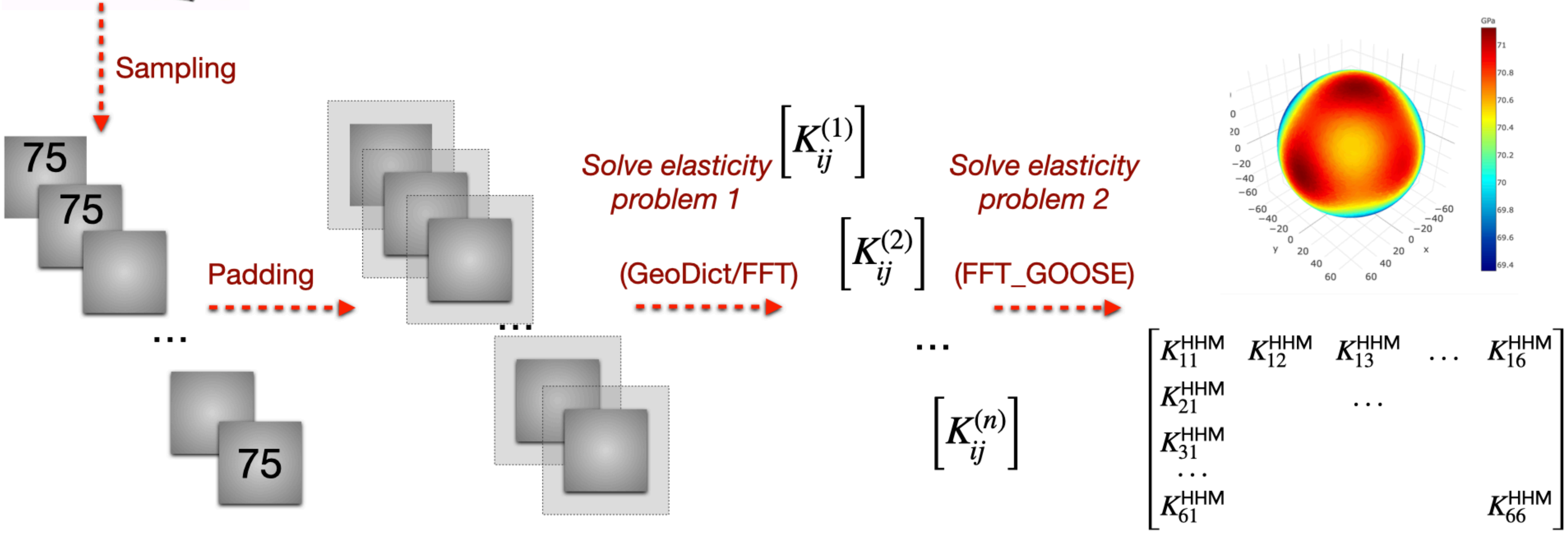
Hierarchical Homogenization (HHM)

HHM solves the elasticity problem twice:

1. On all the subcubes (GeoDict).

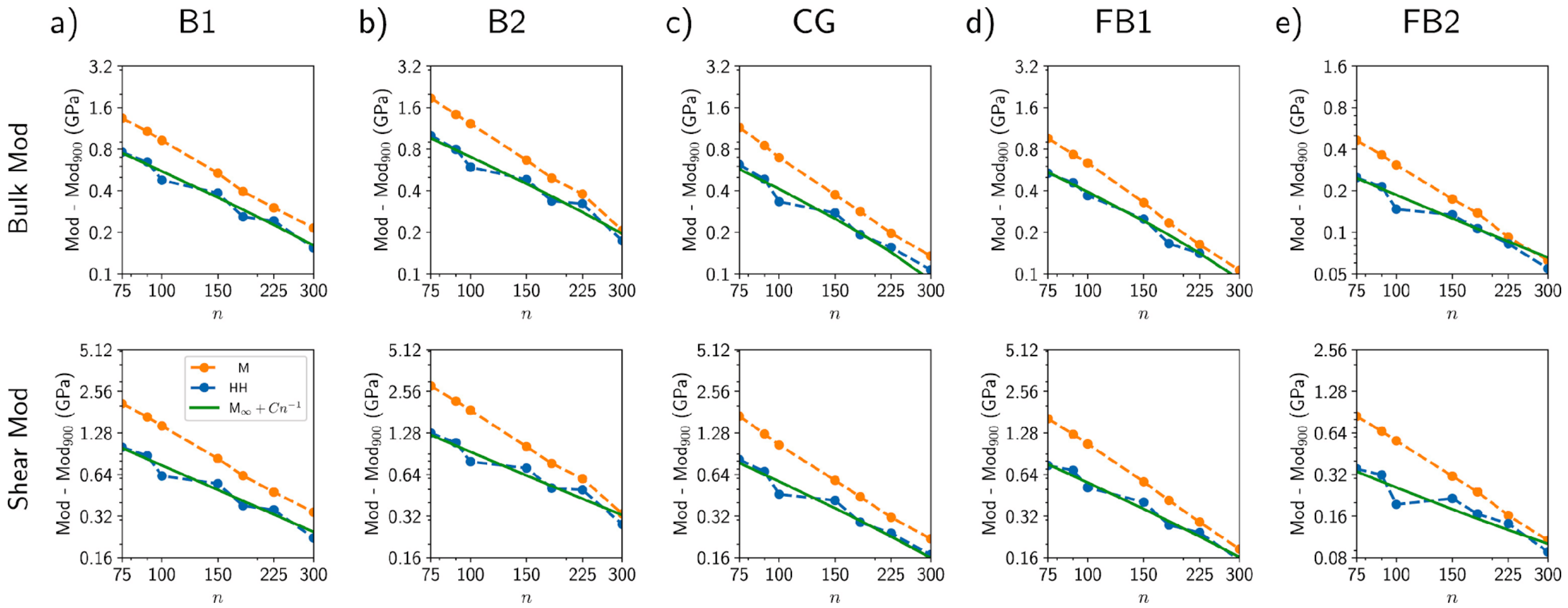
2. On the full rock consisting of homogenized subcubes (FFT_GOOSE/in house).





https://micronano.stanford.edu/research/digital-rock-physics

Hierarchical Homogenization (HHM)



Error analysis:

1. HHM error decays as n^{-1} (n is subcube size).
2. HHM is more accurate than simple average over subcubes.

Admad *et al.*, *JMPS*, 2023, 10.1016/j.jmps.2023.105268

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Gassmann Equation

- Verification with Gassmann Eqn.

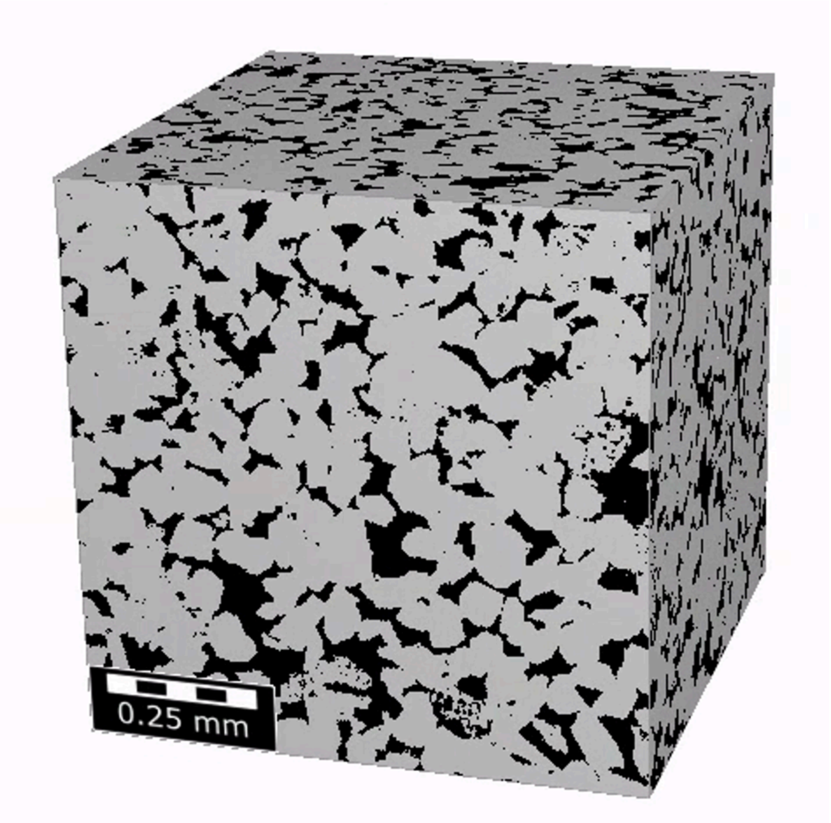
$$K_{sat} = K_{dry} + \frac{\left(1 - \frac{K_{dry}}{K_{min}}\right)^2}{\frac{\phi}{K_{fl}} + \frac{1-\phi}{K_{min}} - \frac{K_{dry}}{K_{min}}}$$

from HHM

Defined properties

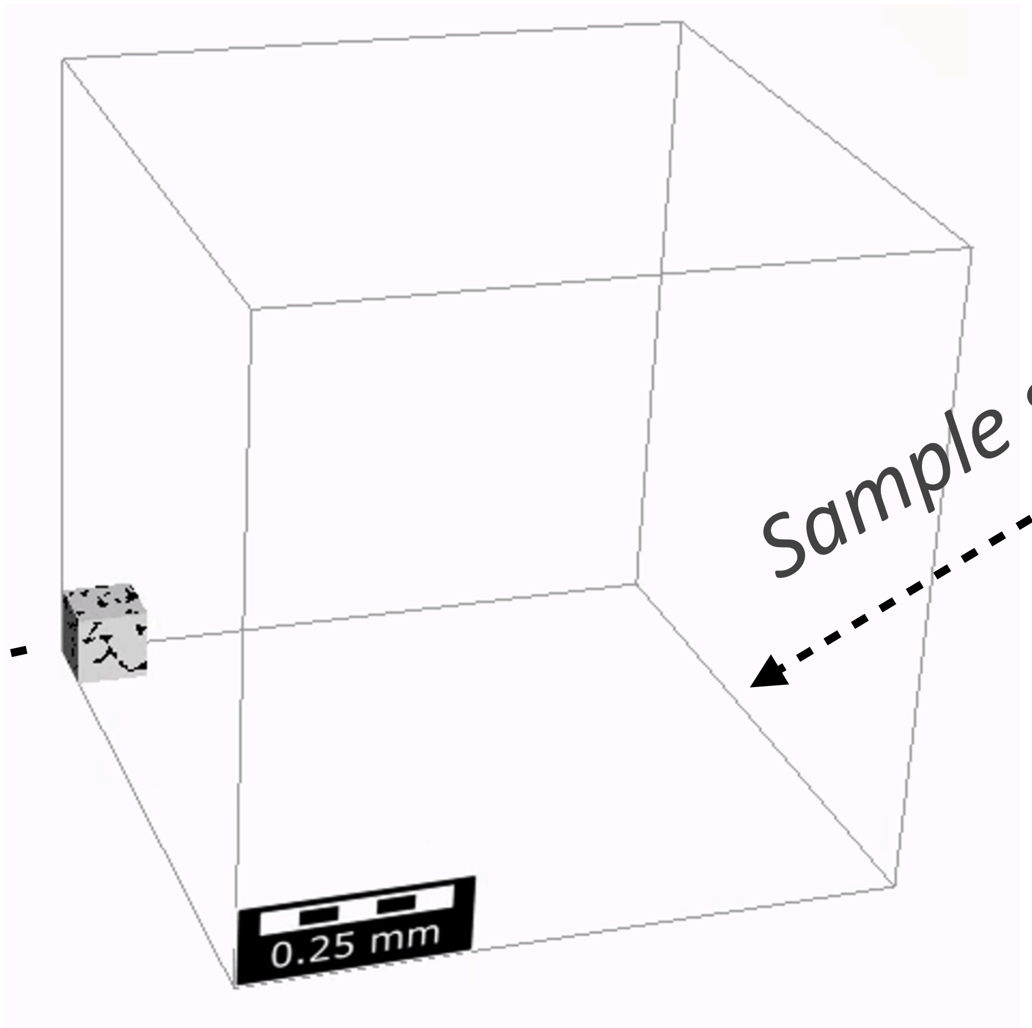
- Subcube size $\mathbb{R}^{100}$ (729 subcubes)

- Full rock size $\mathbb{R}^{900}$ (CG)



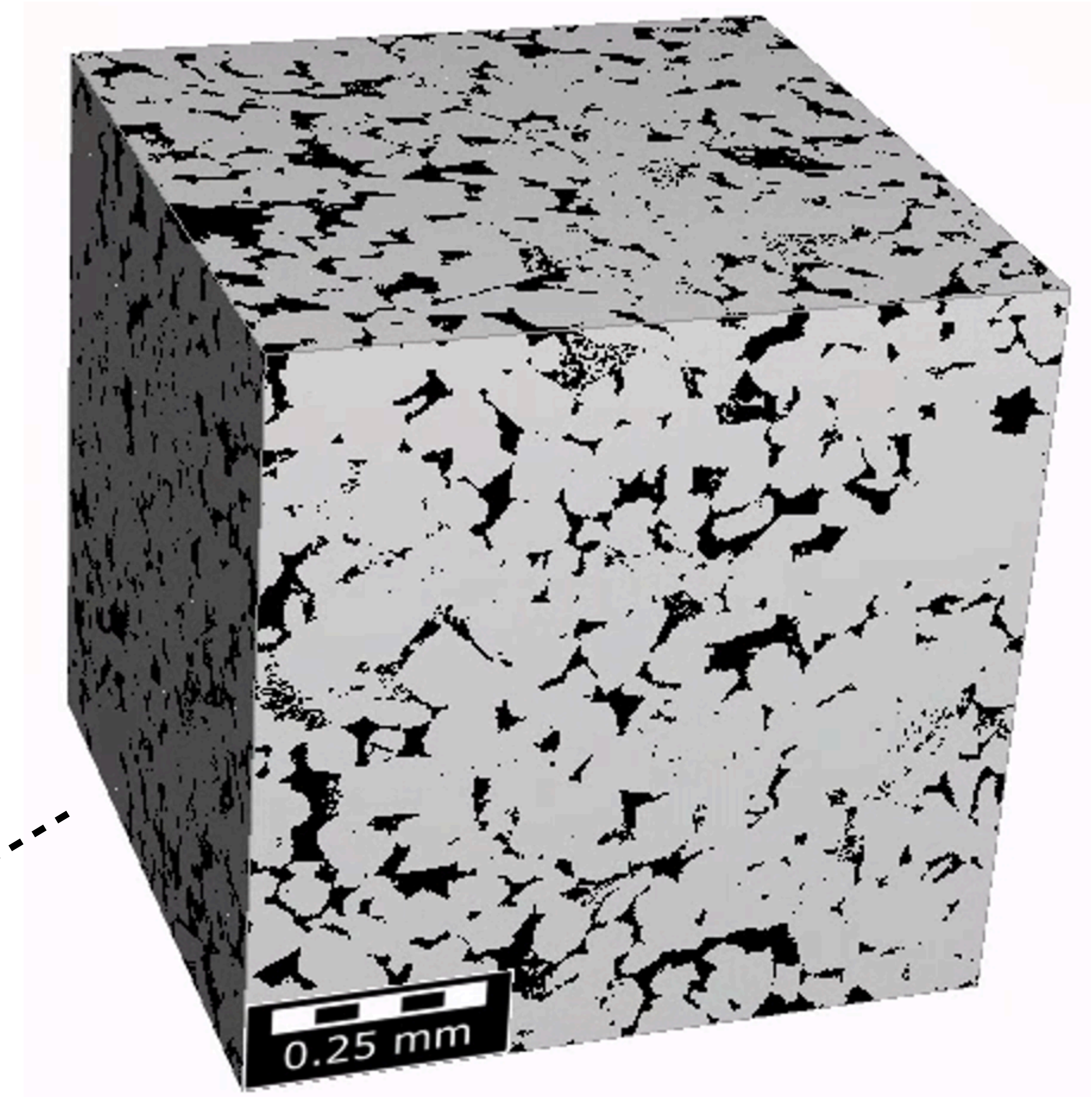
Overall porosity: 22.20%

Segmentation



Sample subcubes

- Full rock size $\mathbb{R}^{900}$ (B1)



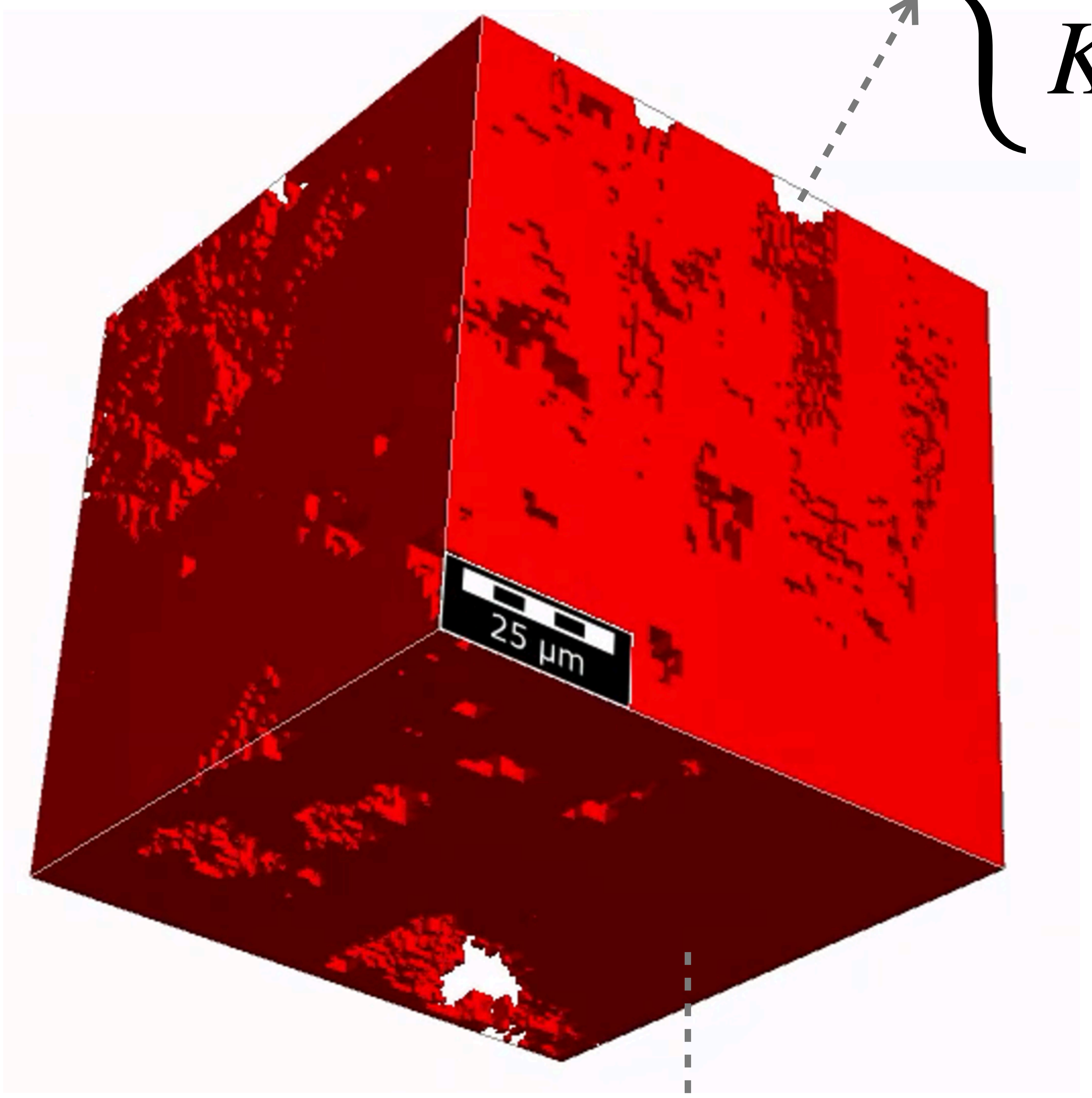
Overall porosity: 16.51%

Stiffness results: https://drive.google.com/drive/folders/1rGHjBuJ7NkypH_oPB2VhzCzuea_YNk5w

Verification of Gassmann Equation (single sub-cube)

- After segmentation

Fluid phase $\left\{ \begin{array}{l} K_{fl} \\ K_{air} \end{array} \right.$



Solid phase K_{min}

Dry Rock

34.2106	4.0096	3.6662	0.4697	-0.2140	-0.0706
4.0096	37.7583	4.0362	1.2725	-0.1819	-0.3833
3.6662	4.0362	36.1864	1.9262	-0.6563	0.3987
0.4697	1.2725	1.9262	15.1915	-0.4254	-0.4918
-0.2140	-0.1819	-0.6563	-0.4254	12.8380	1.1448
-0.0706	-0.3833	0.3987	-0.4918	1.1448	13.6445

#	Voigt	Reuss	Hill
bulk	14.6199	14.4892	14.554584252212372
shear	14.7644	14.5370	14.650697517655548

Saturated Rock

36.8727	6.4455	6.1788	0.3356	-0.1944	-0.0887
6.4455	40.1218	6.3815	1.1324	-0.1395	-0.3994
6.1788	6.3815	38.7191	1.7826	-0.6493	0.4036
0.3356	1.1324	1.7826	15.2621	-0.4419	-0.5021
-0.1944	-0.1395	-0.6493	-0.4419	12.9173	1.1517
-0.0887	-0.3994	0.4036	-0.5021	1.1517	13.7071

#	Voigt	Reuss	Hill
bulk	17.0806	16.9779	17.029246648929053
shear	14.8245	14.6066	14.715562604902011

GeoDict	17.0806 GPa
Gassmann	17.048701629439385 GPa

Relative error: ~ 0.2%

Verification of Gassmann Equation (full rock)

GeoDict Calculation

B1 Rock

- Dry rock homogenized stiffness

```
# Using Hill averged stiffness for each voxel

     54.8578        5.2695        5.1178       -0.0459        0.2531        0.3895
      5.2695       54.8454        5.2193        0.5716       -0.0390        0.3607
      5.1178        5.2193       55.1103        0.5551        0.2730       -0.0307
     -0.0459        0.5716        0.5551       23.6241        0.0451        0.0006
      0.2531       -0.0390        0.2730        0.0451       23.8118        0.1379
      0.3895        0.3607       -0.0307        0.0006        0.1379       23.4797
#        Voigt      Reuss       Hill
bulk     21.7808        21.7717        21.77622652334857
shear     24.1302        24.1089        24.1195828767984
```

- Saturated rock homogenized stiffness

```
# Using Hill averged stiffness for each voxel

     56.9083        7.1754        7.0207       -0.0861        0.2247        0.3473
      7.1754       56.8839        7.1169        0.5105       -0.0534        0.3204
      7.0207        7.1169       57.1436        0.4968        0.2387       -0.0551
     -0.0861        0.5105        0.4968       23.7093        0.0395       -0.0044
      0.2247       -0.0534        0.2387        0.0395       23.8945        0.1294
      0.3473        0.3204       -0.0551       -0.0044        0.1294       23.5635
#        Voigt      Reuss       Hill
bulk     23.7291        23.7225        23.725798477931193
shear     24.2083        24.1884        24.1983623440763
```

$K_{min} = 36$ GPa

Porosity 16.51%

- Gassmann Eqn. K_{sat}

  23.6966 GPa

- Relative error

  ~ 0.1%

- GeoDict calculated K_{sat}

  23.7291 GPa

Verification of Gassmann Equation (full rock)

CG Rock

- Dry rock homogenized stiffness

```
# Using Hill averged stiffness for each voxel

    46.4706      4.6830      4.7652     -0.0067      0.2975      0.1279
     4.6830     43.1597      4.7466      0.3434     -0.0197      0.1129
     4.7652      4.7466     44.9472      0.2855      0.2974     -0.0377
    -0.0067      0.3434      0.2855     18.5605     -0.1021      0.0342
     0.2975     -0.0197      0.2974     -0.1021     19.4321      0.0477
     0.1279      0.1129     -0.0377      0.0342      0.0477     18.9838
#       Voigt     Reuss      Hill
bulk    18.1075       18.0875       18.097490422500584
shear     19.4208       19.3923       19.406565448734984
```

- Saturated rock homogenized stiffness

```
# Using Hill averged stiffness for each voxel

    48.7260      6.9552      6.9579     -0.0258      0.2653      0.1086
     6.9552     45.7082      7.0843      0.3042     -0.0373      0.0964
     6.9579      7.0843     47.3262      0.2494      0.2625     -0.0405
    -0.0258      0.3042      0.2494     18.6368     -0.1086      0.0287
     0.2653     -0.0373      0.2625     -0.1086     19.5027      0.0421
     0.1086      0.0964     -0.0405      0.0287      0.0421     19.0562
#       Voigt     Reuss      Hill
bulk    20.4172       20.4024       20.409818348140647
shear     19.4900       19.4635       19.476782084624787
```

$K_{min} = 36$ GPa

Porosity 22.20%

- Gassmann Eqn. K_{sat}

  20.4012 GPa

- Relative error

  ~ 0.07%

- GeoDict calculated K_{sat}

  20.4172 GPa

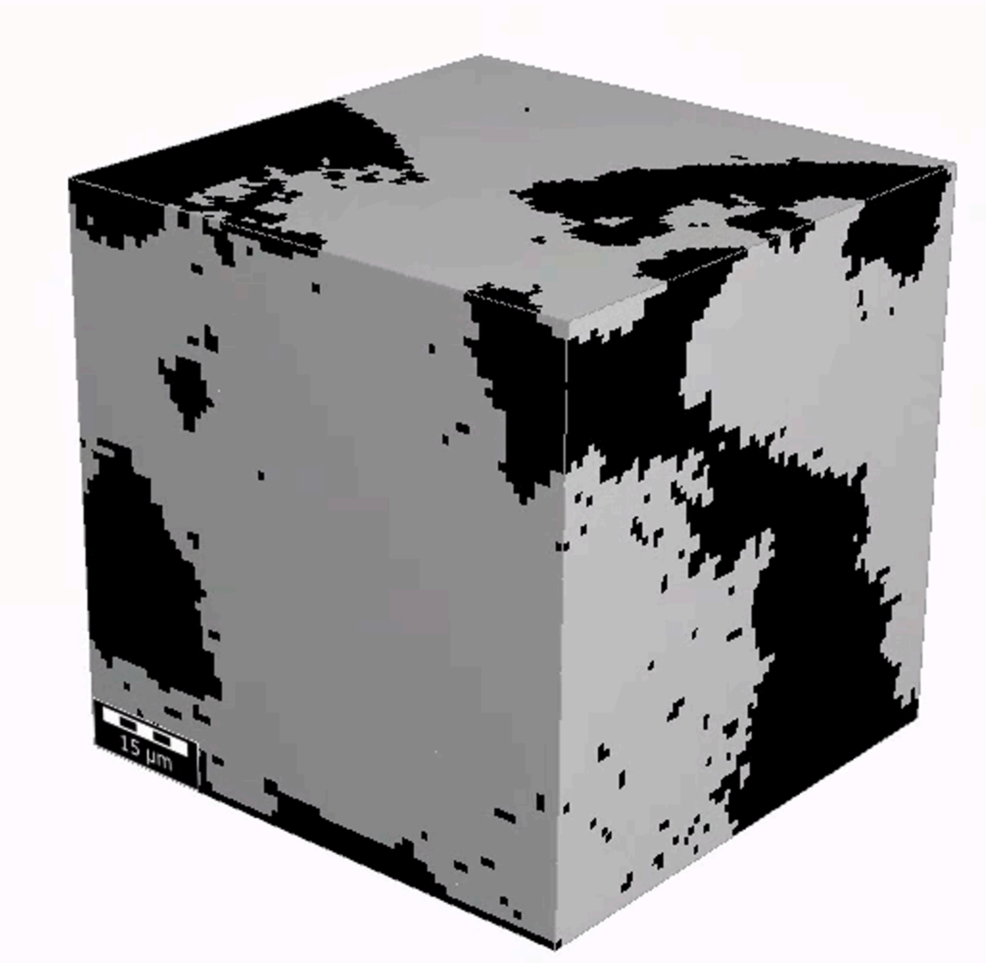
Verification of Gassmann Equation (sub-cube)

FFT-Goose Calculation

B1 Rock

Test: verification of Gassmann Eqn. for B1 rock using FFT-Goose for 1 subcube

- Subcube size: 75
- Porosity: 30.162%



Computation time: ~1 week (MC3 single CPU)

Dry rock

Kdry =

35.2059	3.9546	3.6029	0.4035	-0.7536	1.6616
3.9546	37.0462	4.0203	-0.5947	-0.1428	1.0440
3.6029	4.0203	33.1040	-0.4271	-0.2746	0.4794
0.4035	-0.5947	-0.4271	13.0510	1.1866	-0.4304
-0.7536	-0.1428	-0.2746	1.1866	12.1562	-0.3820
1.6616	1.0440	0.4794	-0.4304	-0.3820	14.7357

Saturated rock

Ksat =

38.2214	6.3627	6.1803	0.4351	-0.7429	1.5590
6.3627	39.7692	6.5027	-0.5897	-0.0956	0.9310
6.1803	6.5027	36.2151	-0.3992	-0.2697	0.3592
0.4351	-0.5897	-0.3992	13.3417	1.1938	-0.4366
-0.7429	-0.0956	-0.2697	1.1938	12.4810	-0.3920
1.5590	0.9310	0.3592	-0.4366	-0.3920	15.0250

Bulk modulus

14.2791 GPa

Gassmann Equation

Bulk modulus

16.7859 GPa

Relative Error

✓ 0.8%

Bulk modulus

16.9219 GPa

Data available via (Rock_75_1.raw): https://drive.google.com/drive/folders/18iJ1EgtYwQV9h3r0uV8FD1fN-fyVc0Qm

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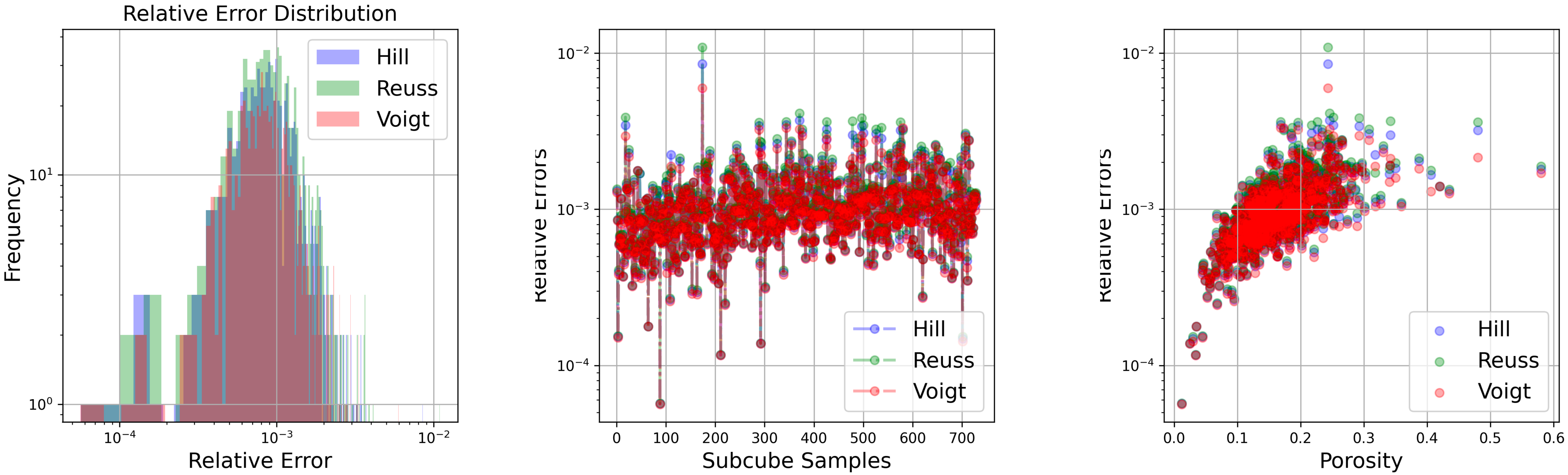
Mechanics of sub-cubes

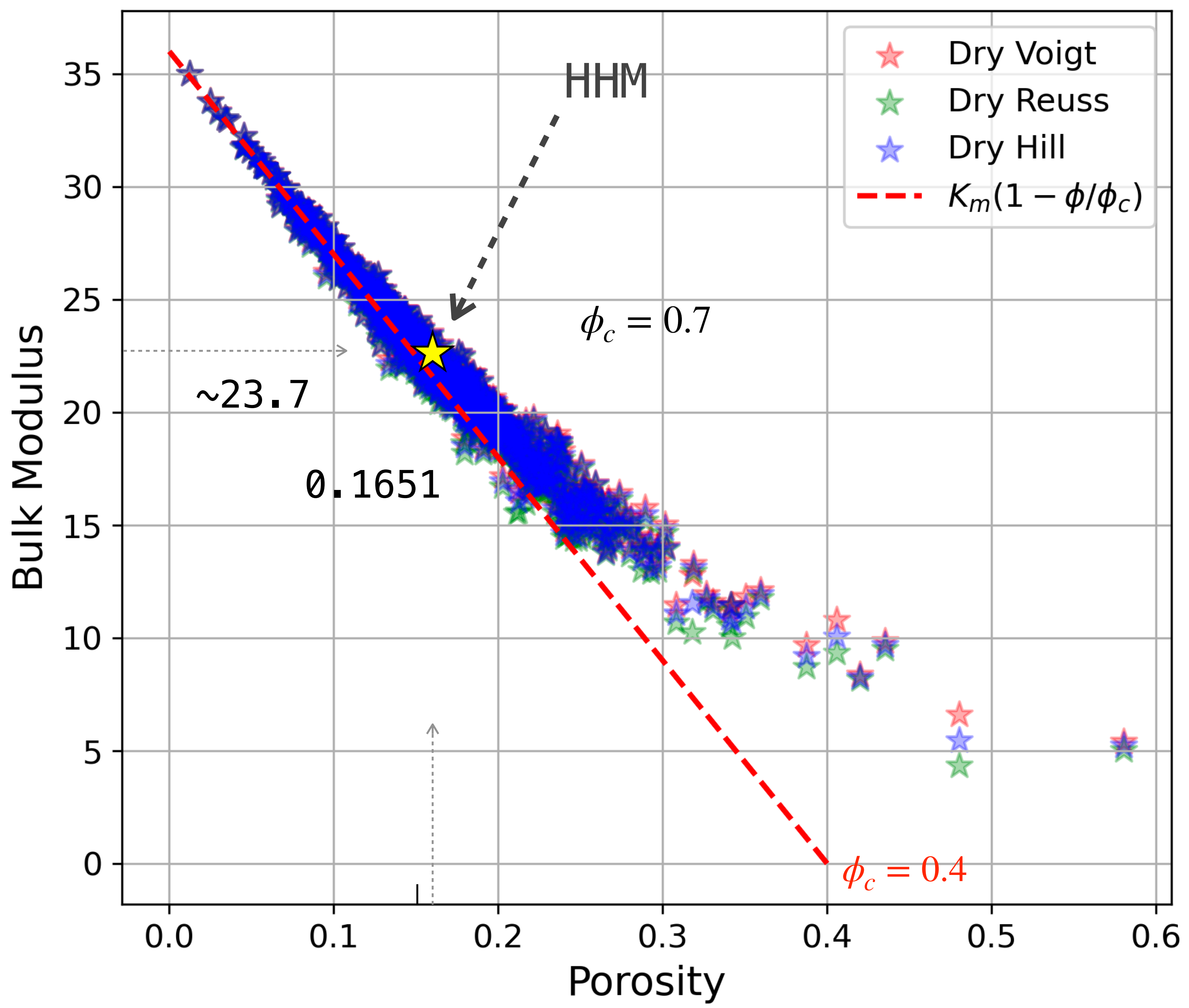
- Different average methods - error analysis

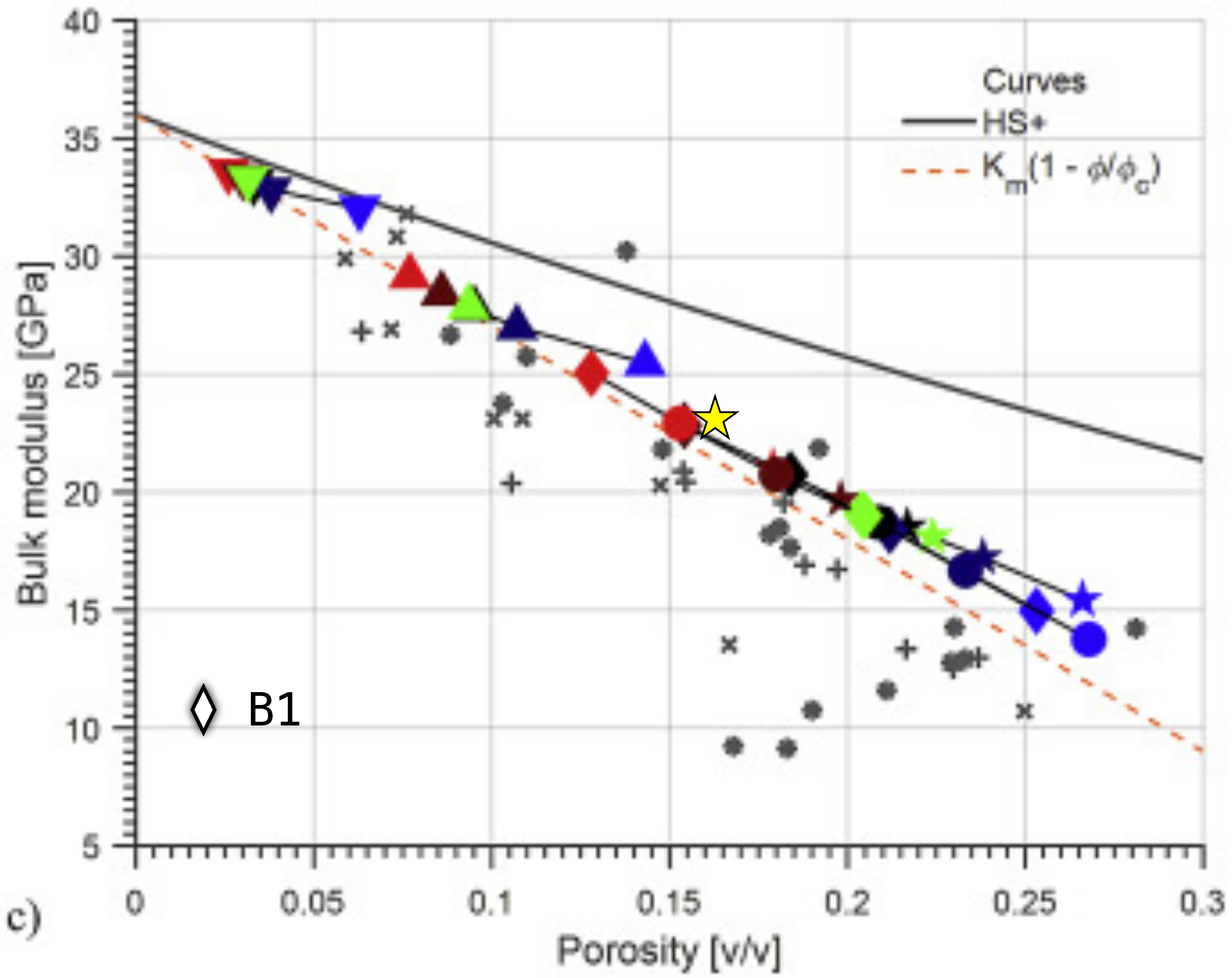


- Voigt method is comparably ore accurate.
- Relative errors positively correlate with porosity.

Mechanics of sub-cubes

- Comparison with theory and results from literature.





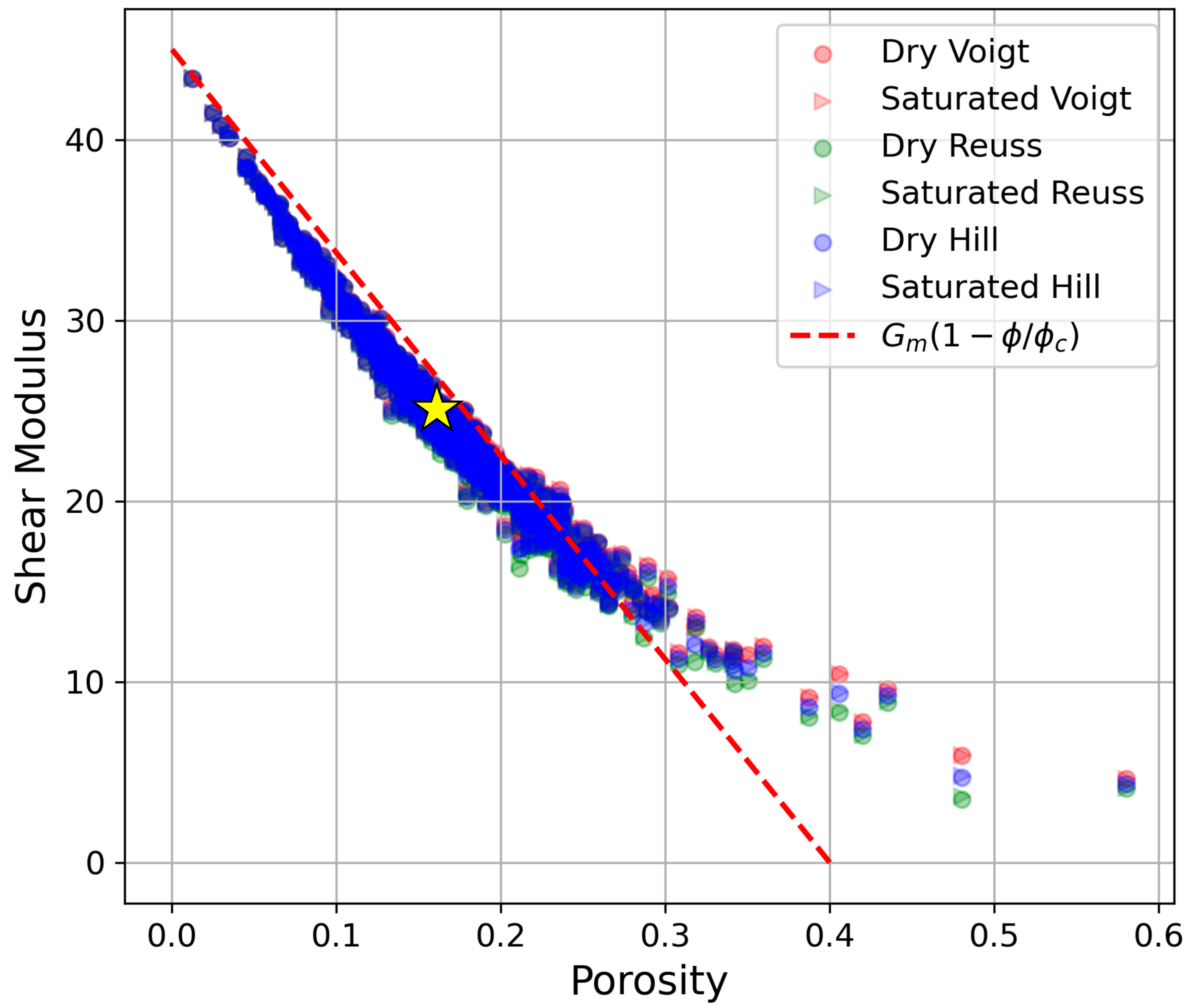
Saxena et al., *Marine and Petroleum Geology*, 2017

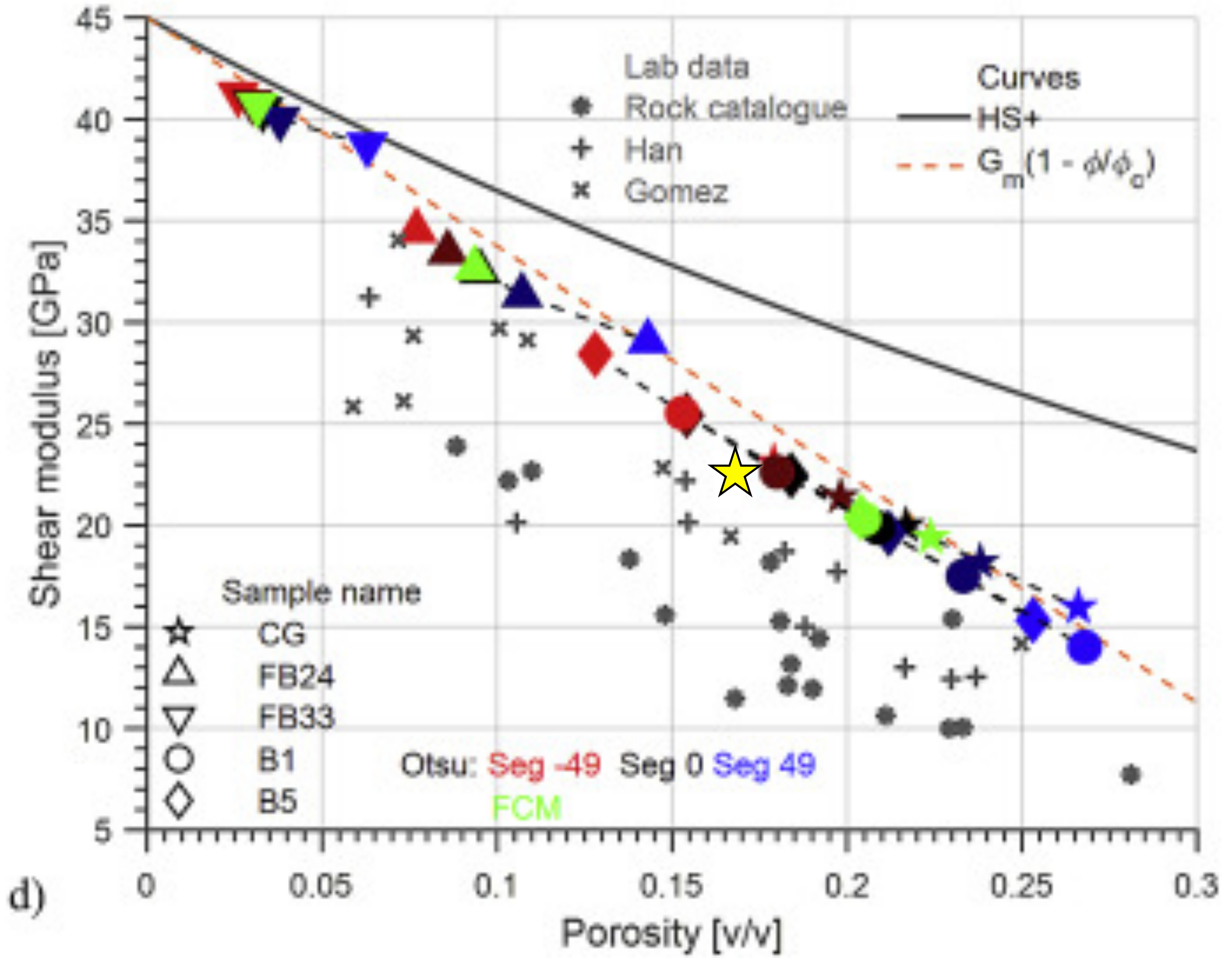
- The bulk modulus-porosity correlation of subcubes agrees well with the linear theory from literature.

Nur *et al.*, *The Leading Edge*, 17(3):357–362, March 1998.

Mechanics of sub-cubes

- Different average methods - error analysis





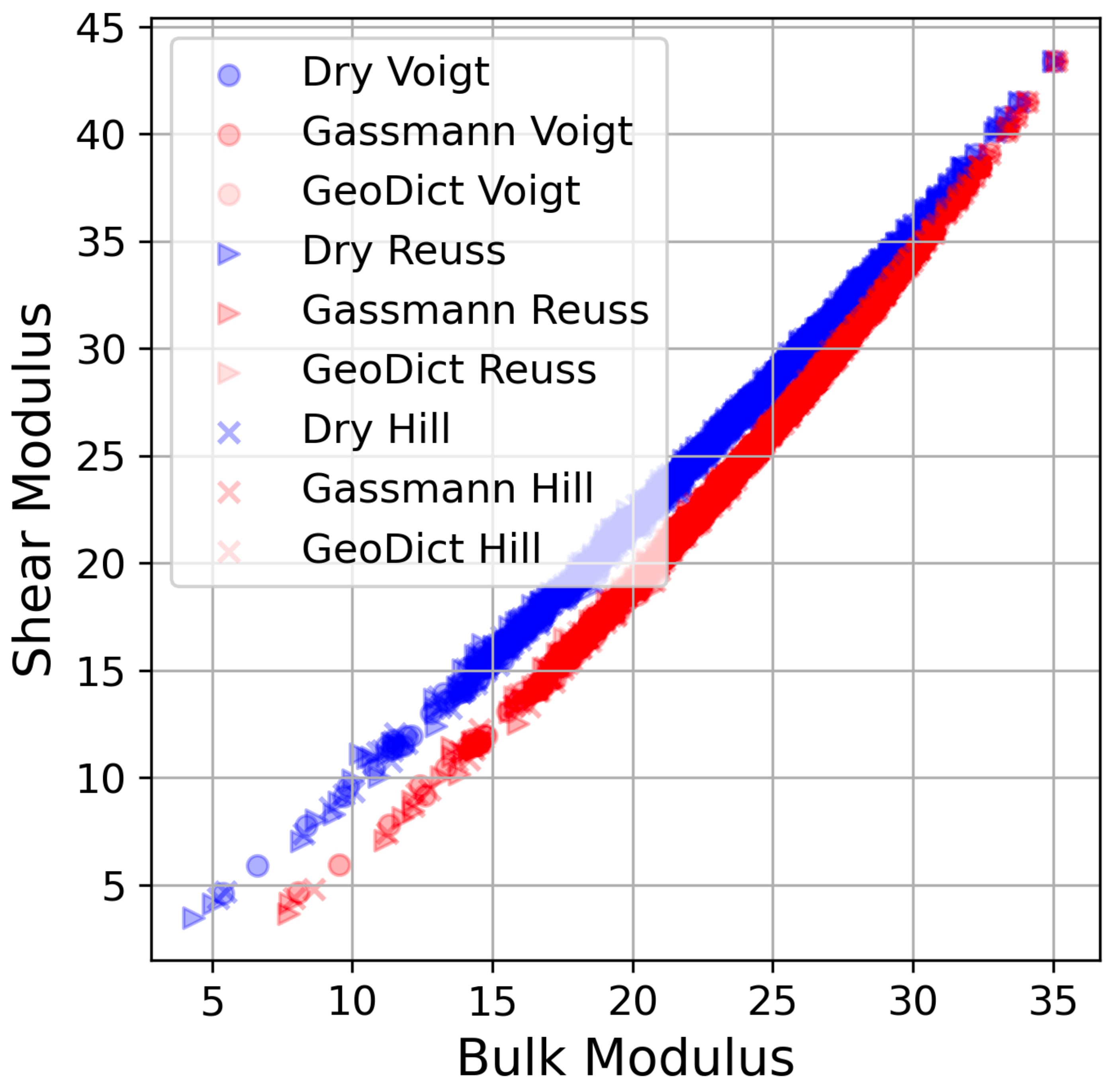
Saxena et al., *Marine and Petroleum Geology*, 2017

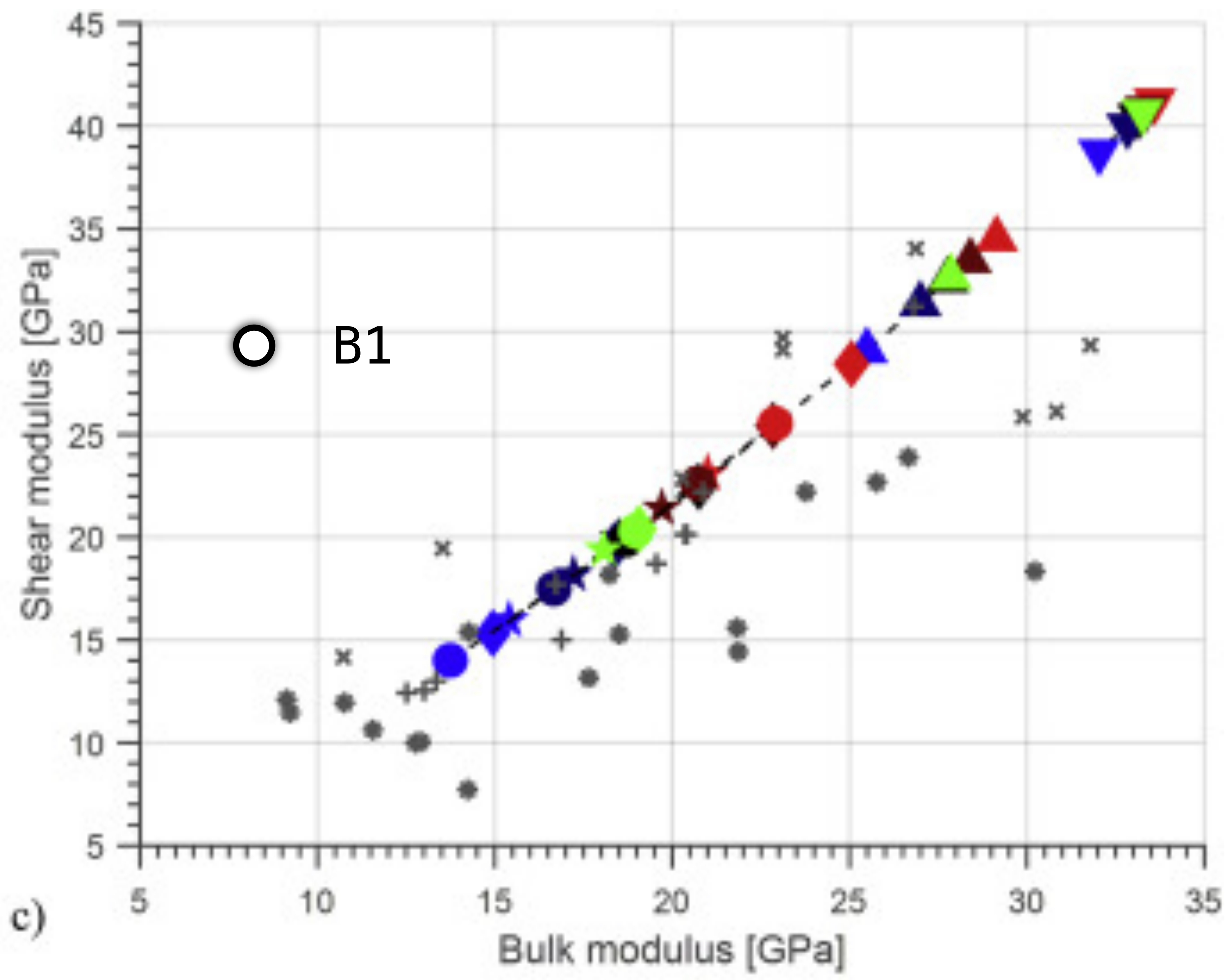
- The shear modulus-porosity correlation of subcubes agrees well with the linear theory from literature.

Nur *et al.*, *The Leading Edge*, 17(3):357–362, March 1998.

Mechanics of sub-cubes

- Different average methods - error analysis





Saxena et al., *Marine and Petroleum Geology*, 2017

- The shear modulus-bulk modulus correlation of subcubes agrees well with the linear theory from literature.
- The dry and saturated digital rocks follow the same trend on the shear-bulk modulus diagram.

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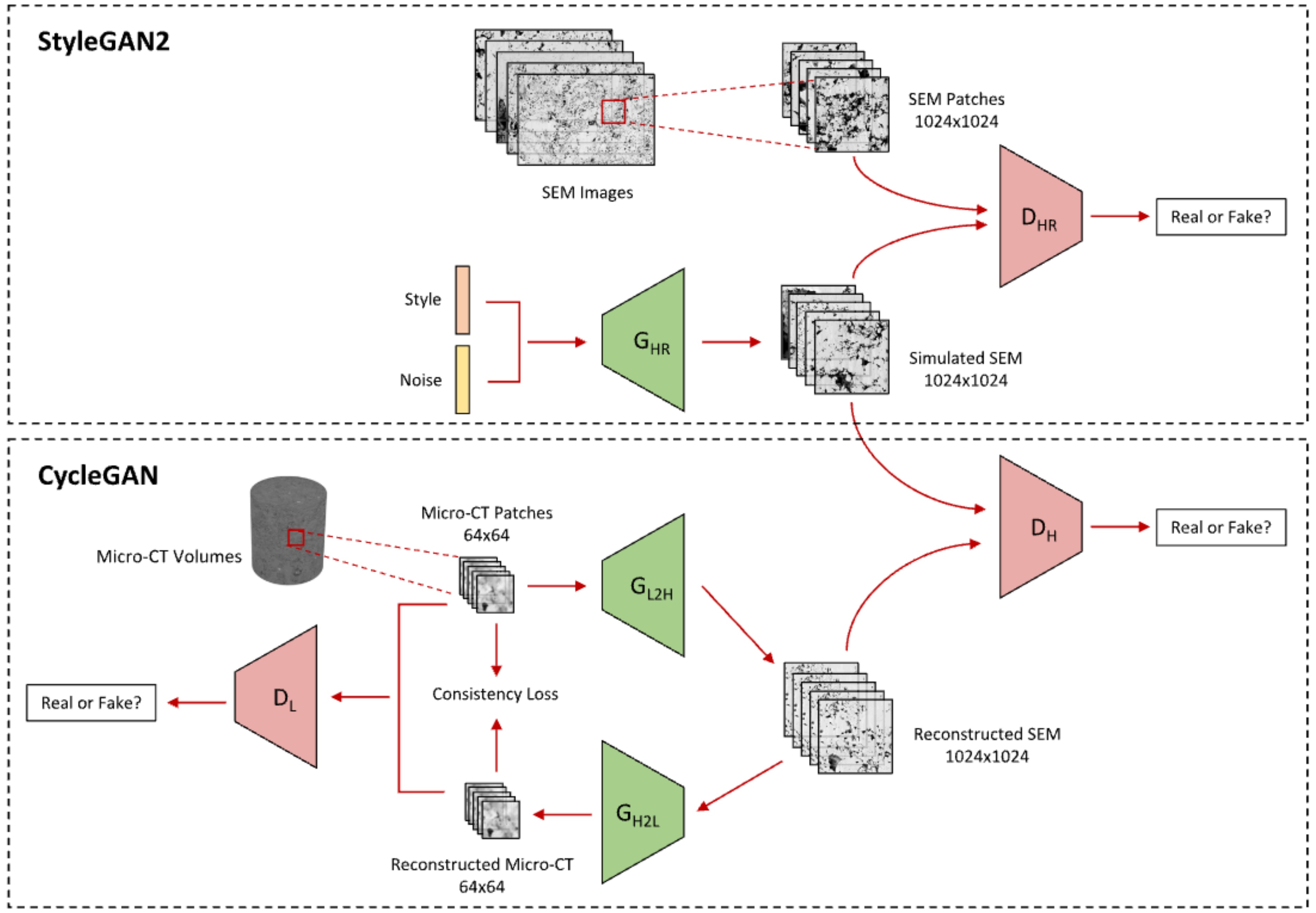
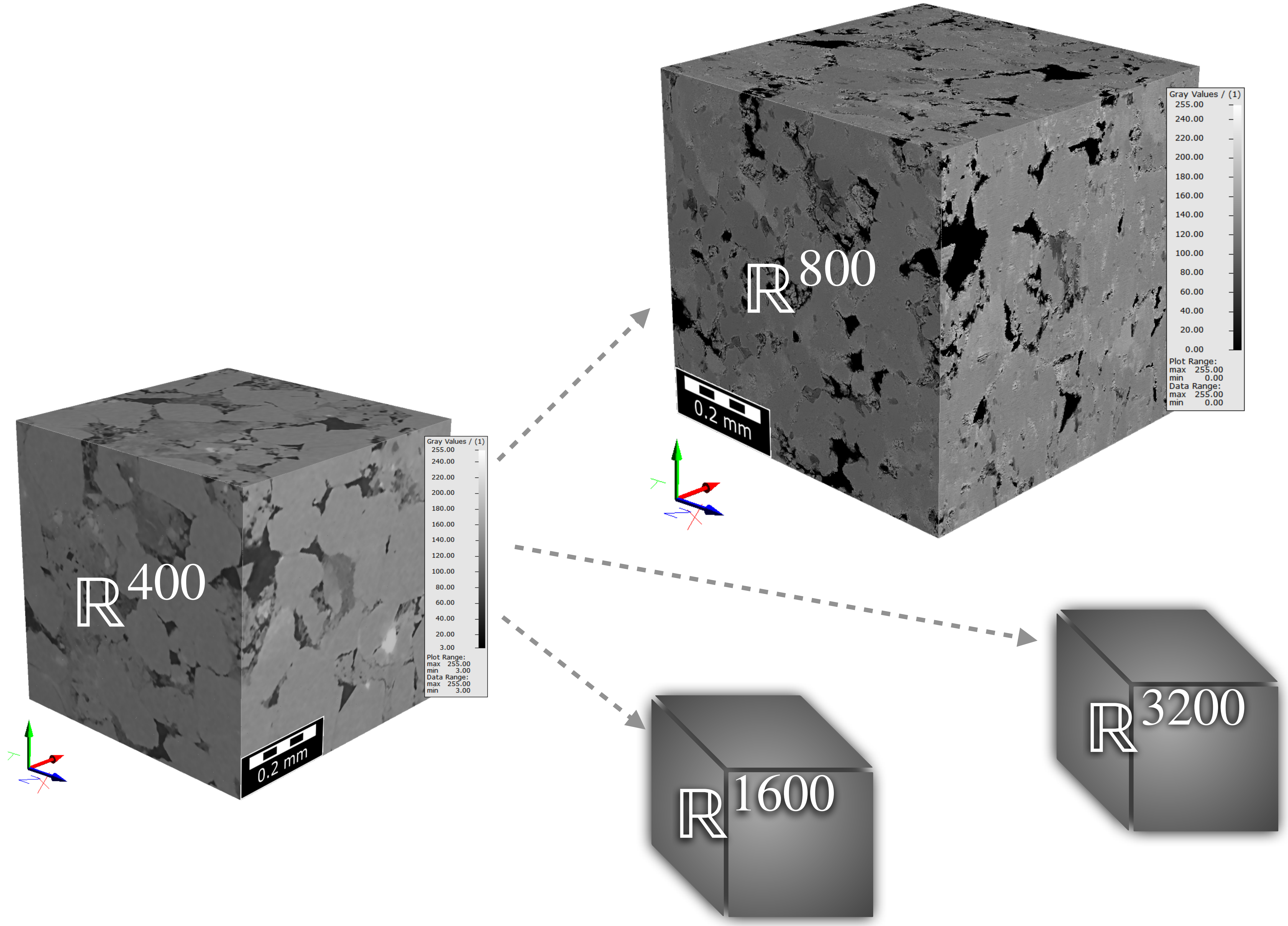
Generation of superresolution via machine learning

1. With prior knowledge and experimentations, porosity correlates negatively with stiffnesses.
2. We want to know how will generative superresolution change the mechanical properties of rocks.
3. Before applying HHM to do the calculation, we want to first check the gray-scale of the generated rocks.

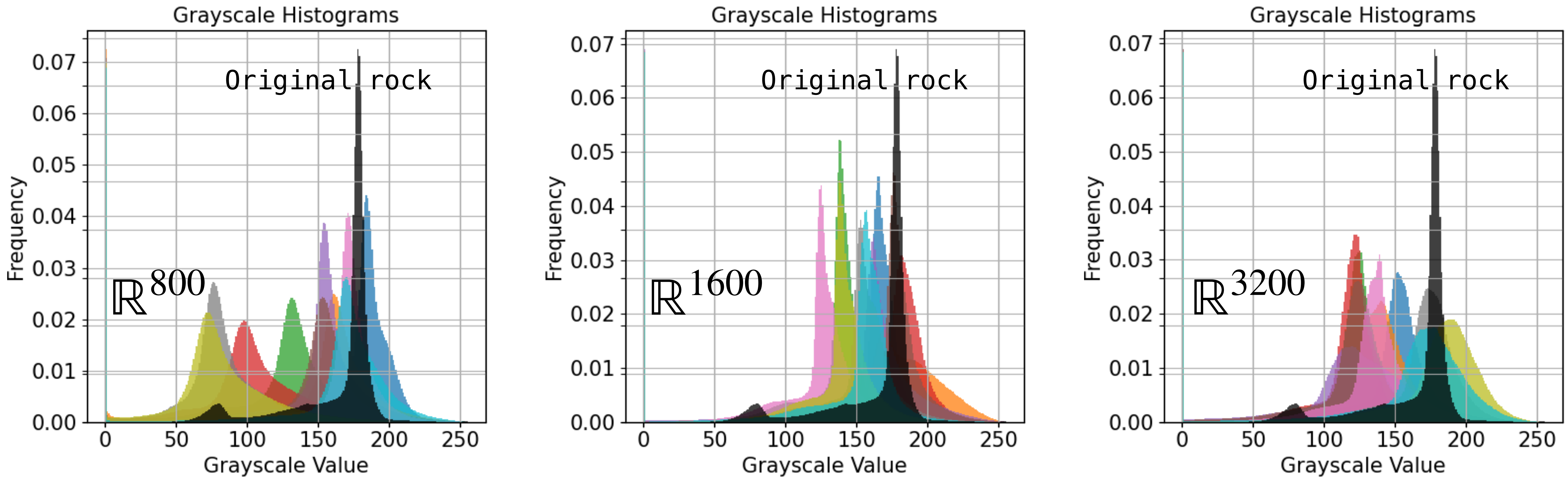




Liu *et al.*, *GRL*, 2022

Gray-scale value distributions of generated rocks

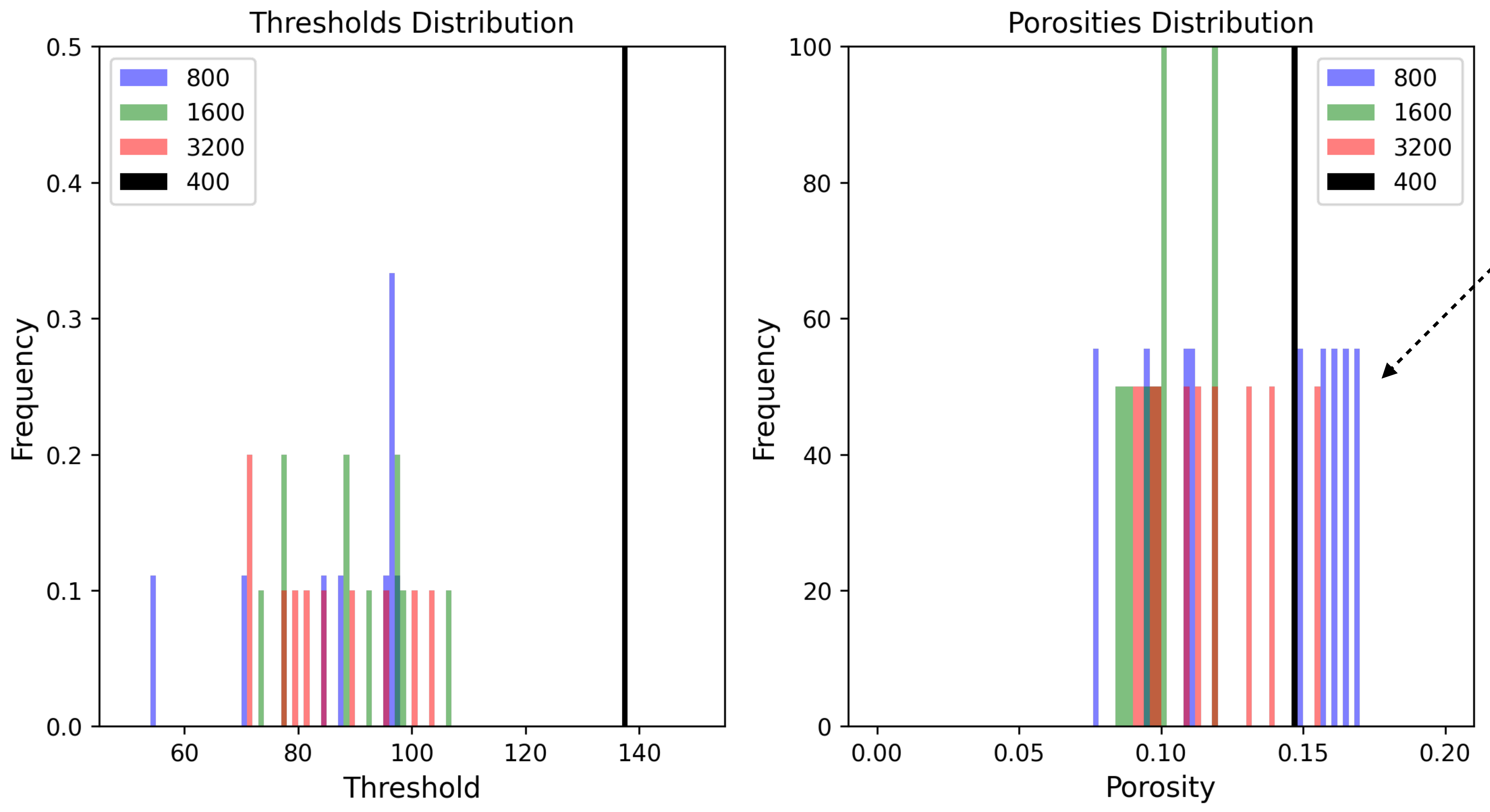
- Comparative analysis on the gray-scale distribution for different superresolution rocks



- $\mathbb{R}^{1600}$ and $\mathbb{R}^{3200}$ rocks' gray-scale values' peaks are centered in ~150, there are a few rocks that have similar peaks with the original $\mathbb{R}^{400}$ rock. $\mathbb{R}^{800}$ rocks' gray-scale values' peaks are more randomly distributed.
- One shall focus on analyzing the rocks that display peaks occur at the similar grayscale value with the original rock (they have similar data distribution).

Porosities and thresholds of generated rocks

- Statistical analysis of the numerical average of the generated rocks



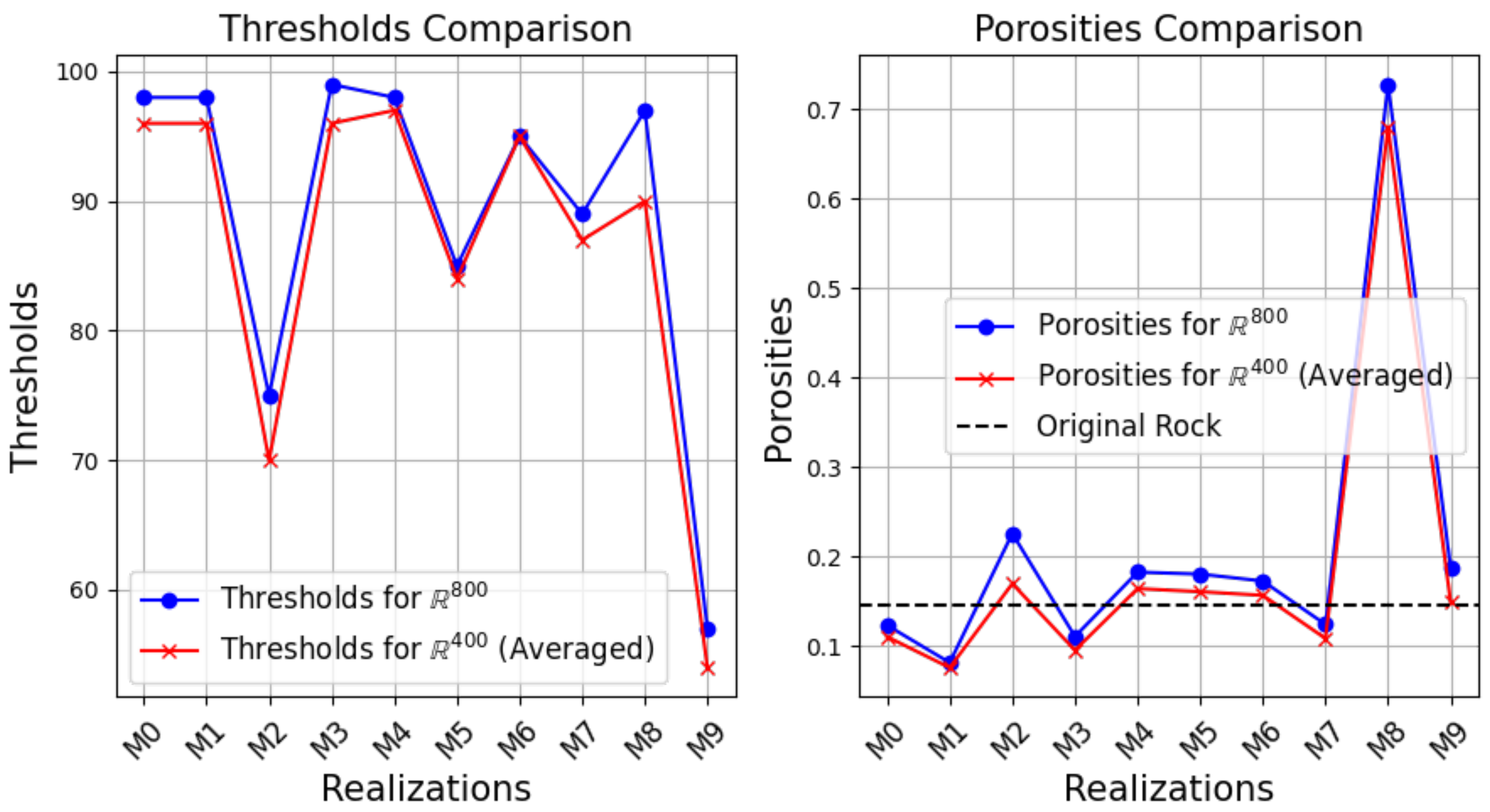
Averaged gray-scale

($\mathbb{R}^{400}$)

- Using the Otsu method, the $\mathbb{R}^{1600}$ rock are less porous after the segmentation based on numerical average of the gray scale.
- The generated gray-scale values are lower.
- Based on our previous analysis, numerical average will decrease the porosity. One may expect that $\mathbb{R}^{1600}$ rock is the most porous (and hence the lowest modulus) before the numerical average.

Effects of numerical average on gray-scale values

- Comparative analysis of the porosity & threshold for $\mathbb{R}^{800}$ rocks before & after numerical average



- Due to the high computational burden for $\mathbb{R}^{1600}$ and $\mathbb{R}^{3200}$ rocks for segmentation from the Otsu method, here I only do a demonstration using the $\mathbb{R}^{800}$ rock to illustrate how numerical averaging decreases the porosity of the rocks.

- The right subfigure also demonstrates that even after the numerical average the generated rocks generally display higher porosities .

Summary

- **General method of HHM and workflow**
- **Benchmarking HHM with Gassmann Equation**
  - GeoDict calculation satisfies Gassmann Eqn. For sub-cube & HHM
  - In-house FFT code calculation satisfies Gassmann Eqn. For sub-cube & HHM
- **Analysis of sub-cube mechanics**
  - Sub-cubes' mechanical properties agrees well with the theory proposed in the literature
- **Statistical analysis of GAN generated rocks**
  - The gray-scale values of the generated rocks are pretty random