Fluid Dynamics Basis & SPH Method

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Equations & Principles

History of Fluid Mechanics

- ARISTOTLE & ARCHIMEDES, *approx.* 300 B.C.: Continuum nature of fluids, Resistance (Drag); Pressure and Body Force.
- LEONARDO DA VINCI, 15 C.: Conservation of Mass (mass flow $(area \times velocity) =$ Const $\rightarrow$ Continuity Equation).
- EDME MARIOTTE & CHRISTIAN HUYGENS, 1673 - 1690: Velocity Squared Law ($force \propto flow\ velocity^2$).
- ISSAC NEWTON, 18 C.: Newtonian Sine-Squared Law (Drag: $D \propto \rho S V^2$; Shear stress: $\tau \propto \frac{dV}{dn}$; Total force: $R = \rho V^2 S \sin^2 \alpha$).
- DANIEL BERNOULLI, 18 C.: Bernoulli Equation of the pressure - velocity relations ($p_1 + \frac{1}{2}\rho V_1^2 = p_2 + \frac{1}{2}\rho V_2^2$).
- HENRI PITOT, 18 C.: Pitot tube for measuring flow velocity of fluids.

Hist. Fld. Mech.

- Leonard Euler, 18 C.: Governing Equations for Inviscous Fluid ($\rho\frac{D\mathbf{u}}{Dt} = \mathbf{F} - \nabla p$).
- William Froude, 18 C.: Dimensional Analysis.
- Navior & Stokes, 19 C.: Governing Equations for Fluid Dynamics / Mechanics ($\rho\frac{D\mathbf{u}}{Dt} = \mathbf{F} - \nabla\sigma$, where $\sigma_{ij} = -p\delta_{ij} + \tau_{ij} \rightarrow$ Substituting the stress tensor we have the N-S Eq.: $\rho\frac{D\mathbf{u}}{Dt} = \mathbf{F} - \nabla p + \mu\nabla^2\mathbf{u}$).
- Osborne Reynolds, 19 C.: Laminar & turbulent flow.
- Kutta & Joukowski, 20 C.: Lift force of airfoil ($L = \rho V\Gamma$, where Γ is the circulation).
- Ludwig Prandtl, 20 C.: Boundary layer theory for fluids.

[Anderson, 2010] & [Bahrami, 2009]

Equations & Principles

I. Continuity Equation (Conservation of Mass).

$$Q = AV = \mathrm{Const} \rightarrow \frac{\partial \rho}{\partial \mathrm{t}} + \nabla \cdot (\rho \mathbf{u}) = 0$$

II. Navier-Stokes Equation (Conservation of Momentum).

$$\rho \left(\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u} \right) = \mathbf{F} - \nabla p + \mu \nabla^2 \mathbf{u}$$

III. Conservation of Energy.

$$\frac{\partial}{\partial t}(\rho e_{\mathrm{total}}) + \nabla \cdot [(\rho e_{\mathrm{total}} + p)\mathbf{u}] = 0$$

IV. Equation of State.

$$p = f(\rho, T), \text{where } p = nRT \text{ for idea gas.}$$

[SciDirec., Fld. Mech.]

Engineering Applications

Aerospace engineering

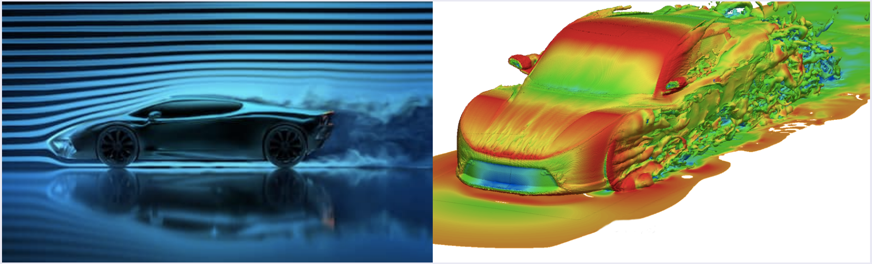
The development of fluid dynamics and related computational methods benefits aerospace engineering drastically.

https://blogs.3ds.com/simulia/aircraft-aerodynamic-performance/

Engineering Applications

Automotive engineering

Both experimental and numerical fluid dynamics knowledge boosted the development of automotive engineering with regards to the aerodynamics and aeronautics involved in body, turbine, wheels, etc.



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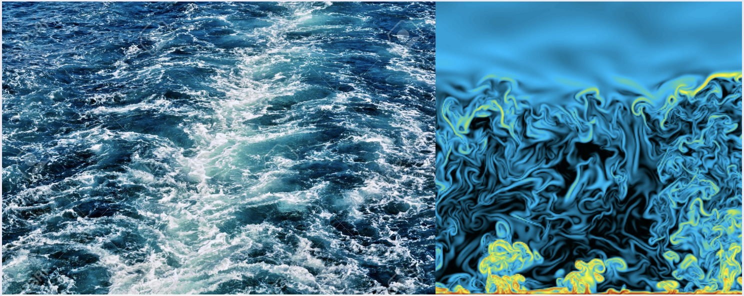
[1] https://engys.com/applications/automotive

[2] https://www.presticebdt.com/3-must-component-for-wind-tunnel-design/

Physics Effects

Turbulence

Turbulence or turbulent flow is fluid motion characterized by chaotic changes in pressure and flow velocity, which is contrast to a laminar flow that occurs when a fluid flows in parallel layers with no disruptions.



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[1] https://www.123rf.com/photo_93088294_background-and-abstract-texture-of-a-turbulence-of-the-made-foam-sea-water-behind-a-vessel-stern.html

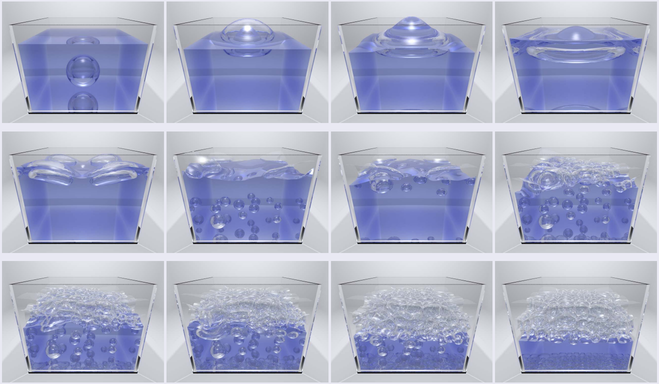
[2] https://www.ipam.ucla.edu/programs/workshops/turbulent-dissipation-mixing-and-predictability/?tab=overview

[3] https://en.wikipedia.org/wiki/Turbulence

Physics Effects

Two-phase flow

Liquid and gas interactions often produce bubbles that stay for a long time without bursting on the surface, making a dry foam structure.



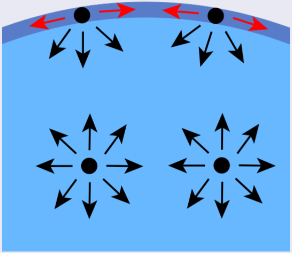
[Kim *et al.*, 2007]

Physics Effects

Surface tension

Surface tension is the tendency of liquid surfaces to shrink into the minimum surface area possible. Surface tension allows insects (e.g. water striders), usually denser than water, to float and slide on a water surface.

$\rightarrow$



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[1] https://www.quora.com/What-is-surface-tension-What-is-the-surface-tension-of-a-water-droplet-and-soap-bubble

[2] https://www.usgs.gov/special-topic/water-science-school/science/surface-tension-and-water?qt-science_center_objects=0#qt-science_center_objects

[3] https://courses.lumenlearning.com/introchem/chapter/surface-tension/

Smoothed-Particle Hydrodynamics

Introduction

Smoothed-particle hydrodynamics is a computational method for simulating continuum mechanics problems, which was developed by Gingold, Monaghan, & Lucy in 1977. It is a meshfree Lagrangian method with wide applications. [Wiki., SPH]

Equations & Principles

Recall the Navier-Stokes equation in terms of material derivatives:

$$\rho \frac{D\mathbf{u}}{Dt} = \mathbf{F} - \nabla p + \mu \nabla^2 \mathbf{u}$$

where $\mathbf{F} = \rho \mathbf{g}$ for fluid in gravity field.

For a single particle i, the governing equation takes the form:

$$\frac{du_i}{dt} = \mathbf{g} - \frac{1}{\rho_i} \nabla p + \frac{\mu}{\rho_i} \nabla^2 u_i$$

Eqs. & Princ.

For any field $F(\mathbf{r})$, [Gingold & Monaghan, 1982] introduced the smoothing kernels $W(\mathbf{r}, h)$:

$$F_s(\mathbf{r}) = \int F(\mathbf{r}) W(\mathbf{r} - \mathbf{r}', h) d\mathbf{r}'$$

where h is the characteristic width of the kernel.

Hence, the approximations to the terms of the Navier-Stokes equations can be discretized as:

$$\rho_i \approx \sum_j m_j W(r - r_j, h)$$

$$\frac{\nabla p_i}{\rho_i} \approx \sum_j m_j \left(\frac{p_i}{\rho_i^2} + \frac{p_j}{\rho_j^2} \right) \nabla W(r - r_j, h)$$

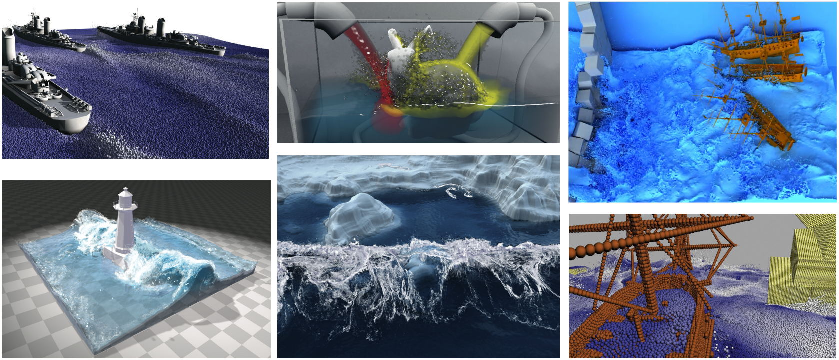
$$\frac{\mu}{\rho_i} \nabla^2 u_i \approx \frac{\mu}{\rho_i} \sum_j m_j \left(\frac{u_j - u_i}{\rho_j} \right) \nabla^2 W(r - r_j, h)$$

[Heirich, 2010]

SPH Applications

Computer graphics

While SPH initially gained popularity for interactive free-surface scenarios, it has emerged to be a fully fledged technique for state-of-the-art fluid animation with versatile effects.

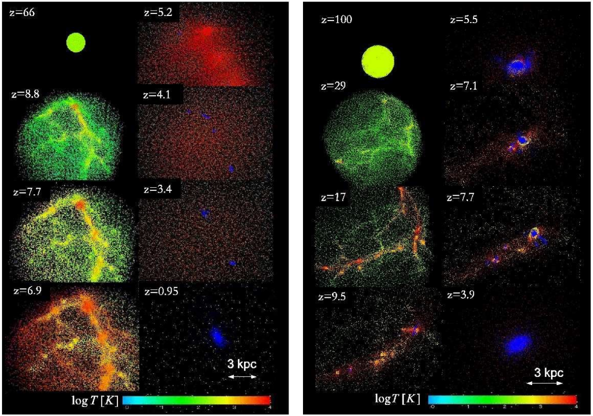


[Ihmsen *et al.*, 2014].

SPH Applications

Astrophysical simulations

Star clusters formed at high-density peaks coalesce with each other in a dissipationless fashion in a dark matter potential, forming a spheroidal system.

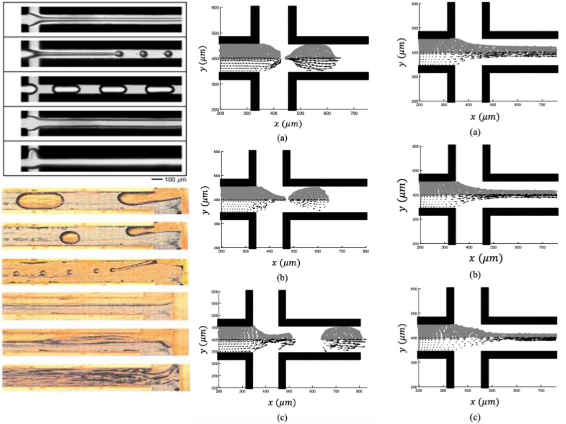


[Susa *et al.*, 2008]

SPH Applications

Microfluidic calculations

A SPH model that facilitates building of relationships between the phase morphologies and fluid dynamics within the phases for the conditions were built.



[Behjati, 2015]

Pros & Cons

Pros

i. Well adapted for simulations involving complex interfaces.
ii. Does not rely on manual meshing (mesh-free).
iii. Provide a large dynamic range in spatial resolution and density.
iv. Excellent conservation properties for both energy, linear & angular momentums.
v. Galilean-invariant and free of any errors from advection alone.

[Springel & Dullemond, 2011]

Cons

i. Shock waves are broadened.
ii. Limited accuracy in multi-dimensional flows, especially for noise generation.
iii. Fluid instabilities across contact discontinuities.
iv. The artificial viscosity is operating at some level also outside of shocks.

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Thanks for Listening!

Any Questions...?